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NUMERICAL PROCEDURES FOR STABILITY STUDIES

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ABSTRACT

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This report presents the numerical procedures used by the Aero-Astrodynamics Laboratory in performing stability analysis for large space vehicles with a more complex control system, and wherein a large number of modes of oscillation must be considered. The modes of oscillation included in the system are (1) bending, (2) translation, (3) pitching, (4) sloshing, and (5) swivel engine. Equations describing the characteristics of the control sensors are included for rate gyros, accelerometers, and angle of attack meter. Two numerical methods for solving the system for its eigenvalues are presented: the characteristic equation and the matrix iteration approach. Finally, a plan for using the procedures in evaluating a vehicle for stability, filter design, and propellant damping is also developed.

Author

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DEFINITION OF SYMBOLS

Symbol	Definition
$A(s)$	transfer function of actuator
a_f	propellant tank radius
a_o	position gain factor
a_{1j}	position rate gain factor
b_o	α-gain factor
e_o	guidance gain factor
e_1	guidance gain factor
F	total thrust
F_s	swivel thrust
$\bar{g}$	vehicle acceleration $\frac{F - X}{M}$
g_{2j}	accelerometer gain factor
I	vehicle moment of inertia about center of gravity
I_{corr}	correction of I due to propellant oscillations
K_1	convenience factor for frequency response is equal to one in stability analysis
$\bar{K}_j$	factor for gain root locus individual channel
$\bar{K}_A$	factor for gain root locus total signal
L_E	distance c.g. of swivel engine to swivel point
M_f	total propellant mass
M_E	swivel engine mass
M_{Bi}	generalized mass of bending modes
M_{SfN}	sloshing mass
$'M$	vehicle mass less engine mass

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
Q	$\frac{\pi D_O^2}{2}$ q aerodynamic pressure coefficient
R	convenience factor for frequency response is equal to zero in stability analysis
S_E	first moment of swivel engine about swivel point
δ_j	artificial phase lag individual channel
δ_A	artificial phase lag total signal
$T_g(s)$	transfer function position gyro filter
$T_{Rj}(s)$	transfer function rate gyro filters
$T_{Aj}(s)$	transfer function accelerometer filters
$\bar{T}(s)$	transfer function α-filter
V	vehicle velocity
X_E	swivel point
$\bar{X}$	distance of x-station to vehicle center of gravity
Y_i, Y_j	normalized bending mode deflection curves
Y_i', Y_j'	slopes of normalized bending mode deflection curves
ζ_{Bi}	percent critical damping (bending)
ζ_{fi}	percent critical damping (slosh)
ζ_{Ri}	percent critical damping (rate gyro)
ζ_{Ai}	percent critical damping (accelerometer)
ζ_E	percent critical damping (swivel engine)
ζ_v	percent critical damping (viscous) α-meter
ζ_M	percent critical damping (mechanical) α-meter
δ_{ij}	Kronecker Delta (0 if $i \neq j$; 1 if $i = j$)

DEFINITION OF SYMBOLS (Cont'd)

Symbol	Definition
θ_E	moment of inertia of swivel engine about swivel point
λ_v	phase lag of local α-meter
ω_v	natural frequency of α-meter
ω_{Ri}	natural frequency of rate gyro
ω_{fi}	natural frequency of sloshing
ω_{Ai}	natural frequency of accelerometer
ω_{Bi}	natural frequency of bending modes
ω_E	natural frequency of swivel engine
λ	ratio local diameter to base diameter
λ'	derivative of λ with respect to x
$C'_{z\alpha}$	local lift coefficient (aerodynamic)
$C_{z\alpha f}$	lift coefficient for fins
A_f	propellant tank or pipe diameter
A'_f	derivative of A_f with respect to x
λ_E	convenience factor
ρ_f	propellant density
$\dot{m}_E$	propellant flow rate
x_1	propellant surface
x_2	propellant pipe end (at engine)
ϵ_n	zeros of $J'(n)$
h_f	propellant height

SUBSCRIPTS

A	accelerometer
B	bending
E	engine
f	sloshing
g	position gyro
R	rate gyro
v	angle-of-attack meter

SUPERSCRIPTS

a	aerodynamics
f	fuel flow

TECHNICAL MEMORANDUM X-53109

NUMERICAL PROCEDURES FOR STABILITY STUDIES

SUMMARY

This report presents the numerical procedures used by the Aero-Astrodynamics Laboratory in performing stability analysis for large space vehicles with a more complex control system, and wherein a large number of modes of oscillation must be considered. The modes of oscillation included in the system are (1) bending, (2) translation, (3) pitching, (4) sloshing, and (5) swivel engine. Equations describing the characteristics of the control sensors are included for rate gyros, accelerometers, and angle of attack meter. Two numerical methods for solving the system for its eigenvalues are presented: the characteristic equation and the matrix iteration approach. Finally, a plan for using the procedures in evaluating a vehicle for stability, filter design, and propellant damping is also developed.

SECTION I. INTRODUCTION

The stability analysis of an elastic space vehicle requires the solution for the eigenvalues of a set of linear, homogeneous differential equations. This set of differential equations describes the dynamics of the space vehicle and the characteristics of the control system. Due to the coupling of the structure and control loop, a control feedback problem is apparent with the structure providing the feedback loop. The final system, then, can be called an electromechanical feedback problem, where conventional control design methods are applicable. For small numbers of degree systems of equations and simple control systems, the solution can be accomplished rather easily. For a more complex control system and a large number of modes of oscillation, this problem is difficult because of the high order of the resulting characteristic equation and the large range in magnitude of the coefficients which yields numerical errors and poor results. This paper presents the procedure and the equations used by Aero-Astrodynamics Laboratory in the solution of this problem. The equations of motion are not derived [1], but are presented in a form needed for solution.

Included in the system are the following modes of oscillation:

bending (μ modes)	(η_μ)
translation	(y)
pitching	(φ)
sloshing (q modes)	(ξ_f)
swivel engine	(β_E).

Additional equations describing the characteristics of the control sensors are included for

rate gyros (r in number)	($\dot{\varphi}_R$)
accelerometers (p in number)	(A_i)
angle-of-attack meter	(α_i).

The control command equation, including transfer functions for all filters, is presented as the final equation. This equation includes the actuator system characteristics.

Two numerical methods for solving the system for its eigenvalues are presented: the characteristic equation and the matrix iteration approach. In all cases, the system is solved for the normalized eigenvectors providing additional information about the system.

Finally, a development plan for using the procedures in evaluating a vehicle for stability, filter design, and propellant damping is presented.

SECTION II. GENERAL APPROACH

A. Basic Equations

The equations of motion [1], the control equation and the equations describing the response characteristics of the sensing elements are homogeneous, linear differential equations. These equations are transformed into a set of homogeneous, linear simultaneous equations by assuming solutions of time dependency in the form e^{st} by which all differential quotients with respect to time are replaced by the complex operator

$$s = \sigma + i\omega. \tag{1}$$

Denoting the coordinates (or unknowns) as X_j ($j = 1, 2, \ldots n$) and the coefficients of X_j as $d_{ij}(s)$ ($i, j = 1, 2, \ldots n$) the set of equations reads

$$
\begin{aligned}
&d_{11}(s)\,X_1 + d_{12}(s)\,X_2 \text{ ---------------- } d_{1n}(s)\,X_n = 0 \\
&d_{21}(s)\,X_1 + d_{22}(s)\,X_2 \text{ ---------------- } d_{2n}(s)\,X_n = 0 \qquad (2) \\
&\text{--} \\
&d_{n1}(s)\,X_1 + d_{n2}(s)\,X_2 \text{ ---------------- } d_{nn}(s)\,X_n = 0.
\end{aligned}
$$

So that a consistent flow of data from other programs needed in equation 2 is kept, the following order and definitions of the X_j and equations should be used throughout:

Number (i =)	Definition	Symbol
$1 \to 6$	Bending modes	η_i
7	Translation	y
8	Rotation	φ
$9 \to 18$	Sloshing	ξ_i
$19 \to 20$	Rate gyros	φ_{Ri}
$21 \to 22$	Accelerometers	A_i
23	Angle-of-attack meter	α_i
24	Swivel Engine	β_E
25	Control	β_c

By using matrix notation, equation (2) can be more conveniently expressed in the form

$$D(s)\ \{X_j\} = 0, \qquad (3)$$

where all of the elements of the matrix D(s) except the n^{th} row has the form

$$d_{ij}(s) = s^2 A_{ij} + s B_{ij} + C_{ij} \tag{4}$$

$$(i = 1 \rightarrow n - 1) \quad (j = 1 \rightarrow n)$$

and the elements for the n^{th} row, which describes the control equation and the filter characteristics, as

$$d_{nj}(s) = K_j T_j(s) \tag{5}$$

$$(j = 1 \rightarrow n).$$

The transfer functions $T_j(s)$ are of the form

$$T_j(s) = \frac{\sum_{n=0}^{n=10} a_{jn} s^n}{\sum_{m=0}^{m=10} b_{jm} s^m} . \tag{6}$$

With the general equations in this form, two basic approaches are available for determining the eigenvalues: (1) matrix iteration and (2) expanding the determinant into a characteristic equation and solving for its roots. Once the roots (eigenvalues) have been determined, it is important to solve for the eigenvectors for additional information. This is done by assuming a value for the engine deflection command angle (X_n or β_c) and solving for the resulting eigenvectors. Usually the eigenvectors are normalized to X_n or β_c equal to 1.

B. Matrix Iteration for Obtaining Eigenvalues

Equation (3) was written as

$$D(s) \{X_j\} = 0$$

and states the transformed equations of the system in matrix form. The problem is to find nontrival values of s and $\{X_j\}$ for which equation (3) is fulfilled. Since the coefficients of D(s) are, in general, polynomials of a higher degree in s, the eigenvalue problem is nonlinear. Starting with an approximate value of s and $\{X_j\}$ it is possible to find a set of linear nonhomogeneous equations for the correction terms which have to be added to the approximate values of s and $\{X_j\}$. By developing equation (3) in a Taylor series at the point s_n and neglecting terms of higher order the iteration procedure [1] is

$$D(s_n)\ \{\Delta X_{jn-1} + D'(s_n)\ \{X_{jn}\}\ \Delta\ s = -\ D(s_n)\ X_{jn} = -y_{jn}, \tag{8}$$

where

$$D'(s_n) = \frac{d\ D(s)}{d\ s}\ .$$

To obtain $\frac{d\ D(s)}{d\ s}$ each element of D(s) is differentiated with respect to s. The numerical procedure for equation (8) is:

Step I:

Insert first approximation of $s_n = s_n^{(0)}$ - obtained from approximate root programs, sponsor load in choice, or the natural frequenies of the modes (Section IV) - into equation (3) and solve for the approximate eigenvectors $\{X_j\}$ by setting $X_n^{(0)}$ equal to one. (Superscript 0 means that the eigenvectors correspond to the approximate root $s^{(0)}$.)

Step II:

Determine the right hand side of equation (8) $\{-y_j\}$ by using the eigenvectors obtained in Step I.

Step III:

Solve equation (8) for Δs. The ΔX_j are not needed, but could be solved for.

Step IV:

Calculate $s^{(1)} = s^{(0)} + \Delta s$.

Step V:

Check s versus a constant (0.001 + i0.001)

a. If either the real or imaginary part is larger than 0.001, begin Step I again with $s^{(1)}$ as $s^{(0)}$ and repeat the above steps.

b. If both real and imaginary parts are smaller than 0.001, the program has converged to an eigenvalue. Using this eigenvalue, compute the ratios of the eigenvectors for the engine deflection β_c as described previously.

Step VI: Print-out results; see Section III.

The iteration procedure is given in the block diagram shown on the following page.

Experience has shown that normalizing the eigenvectors in Step I by assuming x_n equal to 1 does not always give the best convergence. The convergence problem can be relieved by selecting the X_j for normalization to correspond with the eigenvalue which you wish to find. This means that provisions must be made in the program for interchanging rows and columns of equation (4) before starting the iteration procedure with Step I.

C. Characteristic Equation Approach

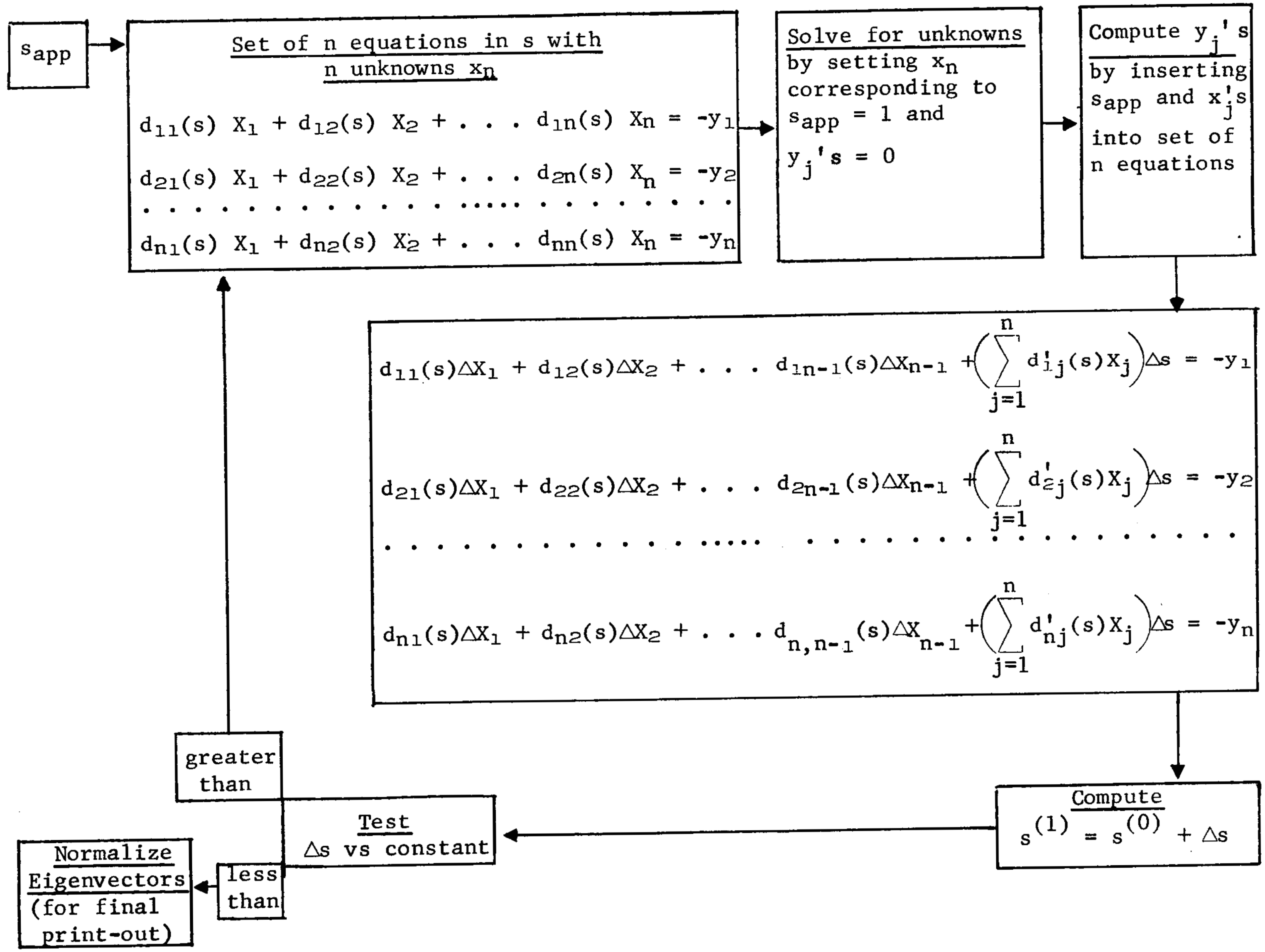
The other approach for solving equation (4) for the eigenvalues is expanding the determinant of the coefficients (D(s)) into a polynomial and solving the polynomial for its roots. Expanding this analytically is much too involved for a large system, if it is possible at all. The solution is done by computing machines, which expand the determinant into a polynomial in s. Since the coefficients are known and certain procedures exist for expanding determinants into polynomials the problem can be solved. However, two restrictions are imposed: The determinant must not be larger than approximately 10 x 10 and the elements must be single polynomials in s, not ratios of polynomials. Since, in general, equation (4) does not meet these two requirements, it must be altered. First, the important modes must be determined, thereby meeting the first condition. To meet the second condition, the determinant must be expanded in minors about the last row since the last row contains ratio of polynomials. The resulting ten determinants must be expanded into polynomials and summed giving the final characteristic equation.

Rewriting equation (4) to accomplish this gives

$$D(s) = \sum_{j=1}^{n} (-1)\ (n-j)\ \bar{T}_j(s)\ \bar{D}^j(s) = 0, \tag{9}$$

where $\bar{D}^j(s)$ is the minors of the determinant D(s), taken on the last row.

BLOCK DIAGRAM OF ITERATION PROCEDURE



Since $\bar{T}_j(s)$ is a ratio of polynomials (to be defined later), it can be written

$$\bar{T}_j(s) = \frac{N_j(s)}{P_j(s)} . \tag{10}$$

Now to get the characteristic equation into a form needed for solution, equation (9) is rationalized and the numerator set equal to zero. The roots of the numerator then become the eigenvalues. D(s) written in this form is

$$D(s) = \sum_{j=1}^{n} \left\{ (-1)^{(n-j)} \bar{D}^{j}(s) \, \bar{N}_j(s) \left(\prod_{k \neq j}^{k} P_k(s) \right) \right\} = 0. \tag{11}$$

Once the eigenvalues have been determined from equation (11), equation (4) must be solved for the normalized eigenvectors with an option to by-pass as described previously in discussing the matrix iteration method.

Two methods for solving the eigenvalue problem have been presented: the matrix iteration method and the characteristic equation.

The matrix iteration method has the advantages that only roots of interest need be obtained, and the size of the system is not critically limited. The disadvantages of this method are that more machine time is required, and gain root locus studies are not well adapted to this method since convergence is poor for high gains.

The characteristic equation approach is advantageous because all roots can be found, less machine time is required, and gain root locus studies offer no adverse convergence problem. However, this approach can be solved only for small systems. Systems giving polynomials higher than 30th order create problems.

D. Frequency Response

In control system design, it is valuable to know the structural transfer function between the swivel thrust and some sensor location, or, in general terms, the frequency response of the system to a sinusoidal forcing function at the control thrust vector point.

This is easily accomplished from equation (3), by letting σ of the assumed solution s be equal to zero, and then assuming various values for ω within the frequency region of interest. For each of these values of ω, equation (3) is then solved for the unknowns X_j by assuming X_n (corresponding to β_c) equal to 1. The thrust force in this case can have unit value. Set R equal to 1 - see definition of elements - and ηK_{1j} equal to zero. The unknowns are in this case complex numbers due to the damping in the system, and must be thus handled.

Using these frequency-dependent unknowns, a structural transfer function can be obtained between a force applied at the swivel point and any sensor location. The equations for computing these various transfer functions are

1. Between thrust and gyro location (See Ref. 1 for basic equations.)

$$\frac{\varphi_i(X)}{F} = \frac{\varphi(\omega)}{F} - \sum_{j=1}^{\mu} \frac{\eta_j(\omega)}{F} Y_j'(X_g). \tag{12}$$

X_g indicates vehicle coordinate for sensor location since the unknowns are complex. A better form for presenting the final results of equation 12 is

$$\frac{\varphi_i}{F}(\omega) = \left|\frac{\varphi_i}{F}(\omega)\right| e^{i <\frac{\varphi_i}{F}(\omega)>} , \tag{13}$$

where $| \;|$ indicates absolute value and $< >$ indicates phase of equation (12).

2. Between the thrust vector and an ideal rate gyro:

$$\frac{\dot{\varphi}_i}{F}(X_R) = i\omega\left(\frac{\varphi}{F} - \sum_{j=1}^{\mu} \frac{\eta_j}{F} Y_j'(X_R)\right). \tag{14}$$

3. Between the thrust vector and an ideal accelerometer:

$$\frac{A_i}{F} = -\omega^2\left(\frac{Y}{F} - \bar{X}_A \frac{\varphi}{F} + \sum_{j=1}^{\mu} \frac{\eta_j}{F} Y_j(X_A)\right). \tag{15}$$

For convenience, equations (14) through (15) can be put in the same form as equation (13).

Other frequency responses can be obtained from equation (3) by solving for the ratios of certain unknowns. For example, φ/β, α/β, etc., are obtained already from the above frequency response, if the actual thrust is used instead of a unit thrust.

SECTION II. BASIC PROCEDURES FOR ANALYZING

The basic question is how to use the procedures presented for conducting a stability analysis. This analysis consists of either a conventional gain root locus, or a phase root locus for a constant gain.

In conventional graphical techniques, the phase root locus is much more difficult to construct than the gain root locus; however, for the numerical procedures presented in this report, these problems are eliminated. This is fortunate since a combination of phase and gain root loci provides considerably more information than available in a root locus diagram (Reference 2, Truxal).

To accomplish the phase root locus, equation (5) is written out as

$$K_n \beta_c = - A(s) \sum_{j=1}^{n-1} K_j T_j(s) X_j. \tag{16}$$

Since this is the control equation written in general form with all the transfer functions of the various loops, a phase locus can now be computed for the total system, or for each control loop separately, by writing the K_j's in the form

$$K_j = \bar{K}_j e^{is_j}. \tag{17}$$

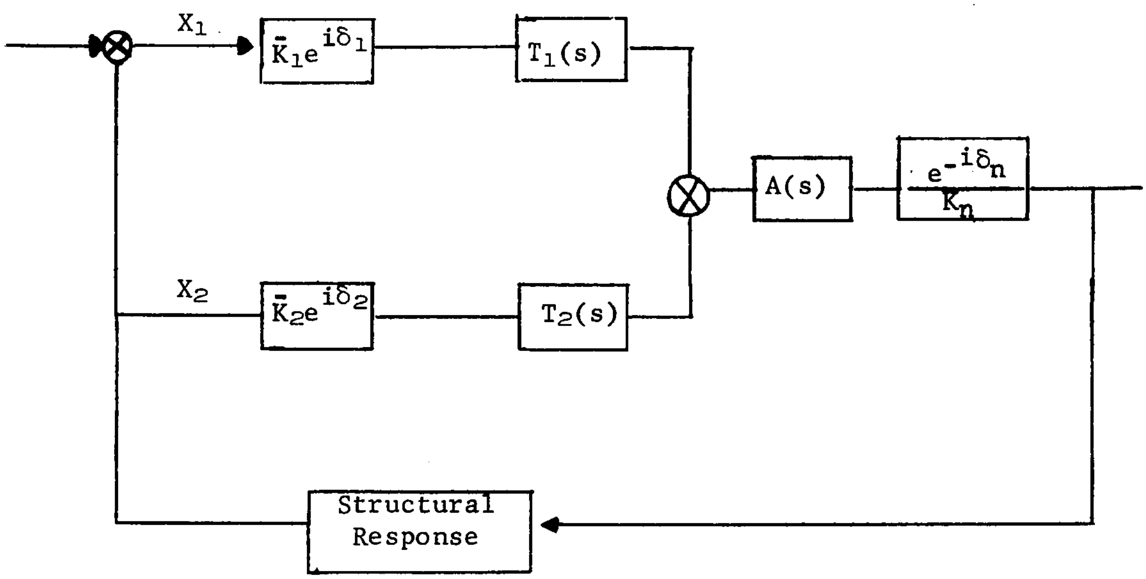
Equation (16) then becomes

$$\bar{K}_n e^{i\delta_n} \beta_c = - A(s) \sum_{j=1}^{n-1} T_j(s) \bar{K}_j e^{i\delta_j} X_j. \tag{18}$$

Now $\bar{K}_j$ contains the open loop gains of each loop as parameter and is kept constant as the phase δ_j is varied and the roots of equation (3) computed for each value of δ_j.

It is clear that a total phase root locus of the system is accomplished by varying δ_n and the effect of varying the phase of an individual loop is accomplished by varying the appropriate δ_j.

The following block diagram depicts the system and procedure.



The matrix iteration procedure works much better for the phase root locus, since the complex elements introduced into the control equation does not cause any convergent problems. This is not always true in the root solving procedures for polynomials.

SECTION III. EQUATIONS OF ELEMENTS

The coefficients of $S^2(A_{ij})$ are

Bending modes: $(i = 1 \rightarrow 6)$

$j = 1 \rightarrow 6$

$$A_{ij} = \frac{Q}{V^2} A_{ij}^{(a)} - Y_j'(X_E) \left[S_E Y_i(X_E) + \theta_E Y_i'(X_E) \right]$$

$$- Y_j(X_E) \left[M_E Y_i(X_E) + S_E Y_i'(X_E) \right] - M_{Bi}\, \delta_{ij}$$

$j = 7$

$$A_{i7} = -\frac{Q}{v^2} G_i - M_E Y_i(X_E) - S_E Y_i'(X_E)$$

$j = 8$

$$A_{i8} = \frac{Q}{v^2} G_i + M_E Y_i(X_E) \left[(X_E + L_E) \right] + Y_i'(X_E) \left[(\theta_E + S_E X_E) \right]$$

$j = 9 \rightarrow 18$

$$A_{ij} = - Y_i(X_f) M_{fj}$$

$j = 19 \rightarrow 23$

$$A_{ij} = 0$$

$j = 24$

$$A_{i24} = S_E Y_i(X_E) + \theta_E Y_i'(X_E)$$

$j = 25$

$$A_{i25} = 0$$

Translation (i = 7):

$j = 1 \rightarrow 6$

$$A_{7j} = \frac{-Q}{v^2} G_j - M_E Y_j(X_E) - S_E Y_j'(X_E)$$

$j = 7$

$$A_{77} = \frac{-Q}{v^2} J_o - M - M_E$$

$j = 8$

$$A_{78} = \frac{Q}{v^2} J_1$$

$j = 9 \rightarrow 18$

$$A_{7j} = - M_{fj}$$

$j = 19 - 23$

$$A_{7j} = 0$$

$j = 24$

$$A_{7,24} = S_E$$

$j = 25$

$$A_{7,25} = 0$$

<u>Rotation (i = 8):</u>

$j = 1 \rightarrow 6$

$$A_{8j} = \frac{Q}{V^2} \bar{G}_j + \theta_E Y_j'(X_E) + \dot{S}_E \bar{X}_E Y_j'(X_E) + Y_j(X_E) (M_E \bar{X}_E + S_E)$$

$j = 7$

$$A_{87} = \frac{Q}{V^2} J_1$$

$j = 8$

$$A_{88} = - I + I_{corr} - \frac{Q}{V^2} J_2$$

$j = 9 \rightarrow 18$

$$A_{8j} = \bar{X}_{fj} M_{fj}$$

$j = 19 \rightarrow 23$

$$A_{8j} = 0$$

$j = 24$

$A_{8,24} = -\theta_E - \bar{X}_E \; S_E$

$j = 25$

$A_{8,25} = 0$

<u>Sloshing ($i = 9 \rightarrow 18$):</u>

$j = 1 \rightarrow 6$

$A_{ij} = Y_j(X_{fi})$

$j = 7$

$A_{i7} = 1$

$j = 8$

$A_{i8} = - \bar{X}_{fi}$

$j = 9 \rightarrow 18$

$A_{ij} = 0$

$i \neq j$

$A_{ij} = 1$

$i = j$

$j = 19 \rightarrow 25$

$A_{ij} = 0$

<u>Rate Gyro (i = 19, 20):</u>

$j = 1 \rightarrow 18$

$A_{ij} = 0$

$j = 19,\ 20$

$A_{ij} = 0$
$i \neq j$

$A_{ij} = 1/\omega_{Ri}^2$
$i = j$

$j = 21 \rightarrow 25$

$A_{ij} = 0$

<u>Accelerometers (i = 21, 22):</u>

$j = 1 \rightarrow 6$

$A_{ij} = - Y_{Aj}(X_{Ai})$

$j = 7$

$A_{i7} = - 1$

$j = 8$

$A_{i8} = + \bar{\bar{X}}_{Ai}$

$j = 9 \rightarrow 20$

$A_{ij} = 0$

$j = 21,\ 22$

$A_{ij} = 0$
$i \neq j$

$A_{ij} = 1/\omega_{Ai}^2$
$i = j$

$j = 23 \rightarrow 25$

$A_{ij} = 0$

<u>Angle-of-Attack meter (i = 23):</u>

$j = 1 \rightarrow 6$

$A_{23,j} = 1/\omega_v^2\ Y'_j\ (X_v)$

$j = 7$

$A_{23,7} = 0$

$j = 8$

$A_{23,8} = -\ 1/\omega_v^2$

$j = 9 \rightarrow 22$

$A_{23,j} = 0$

$j = 23$

$A_{23,23} = 1/\omega_v^2$

$j = 24,\ 25$

$A_{23,j} = 0$

Swivel Engine (i = 24)

$j = 1 \rightarrow 6$

$$A_{24,j} = + \frac{1}{\omega_E^2} \left[\frac{S_E}{\theta_E} Y_j(X_E) + Y_j'(X_E) \right]$$

$j = 7$

$$A_{24,7} = \frac{1}{\omega_E^2} \; \frac{S_E}{\theta_E}$$

$j = 8$

$$A_{24,8} = - \frac{1}{\omega_E^2} \left[\frac{S_E}{\theta_E} \bar{X}_E + 1 \right]$$

$j = 9 \rightarrow 23$

$$A_{24,j} = 0$$

$j = 24$

$$A_{24,24} = - 1/\omega_E^2$$

$j = 25$

$$A_{24,25} = 0$$

The coefficients of $S(B_{ij})$ are

Bending Modes ($i = 1 \rightarrow 6$):

$j = 1 \rightarrow 6$

$$B_{ij} = \frac{Q}{V} B_{ij}^{(a)} - B_{ij}^{(f)} - \delta_{ij} \, 2\zeta_{Bi} \, \omega_{Bi} \, M_{Bi}$$

$j = 7$

$$B_{i7} = -2\frac{Q}{v}D_i$$

$j = 8$

$$B_{i8} = 2\frac{Q}{v}(\bar{D}_i + G_i) + B_{i8}^{(f)}$$

$j = 9 \rightarrow 23$

$$B_{ij} = 0$$

$j = 24$

$$B_{i24} = B_{i24}^{(f)}$$

$j = 25$

$$B_{i25} = 0$$

Translation (i = 7):

$j = 1 \rightarrow 6$

$$B_{7j} = -\frac{2Q}{v}(D_j + H_j) - B_{7j}^{(f)}$$

$j = 7$

$$B_{77} = -2\frac{Q}{v}F_o$$

$j = 8$

$$B_{78} = 2\frac{Q}{v}(F_1 + J_o) + B_{78}^{(f)}$$

$j = 9 \rightarrow 23$

$$B_{7j} = 0$$

$j = 24$

$$B_{7,24} = 2\bar{b}$$

$j = 25$

$$B_{7,25} = 0$$

Rotation (i = 8):

$j = 1 \rightarrow 6$

$$B_{8j} = \frac{2Q}{V} (\bar{D}_j + \bar{H}_j) + B_{8j}^{(f)}$$

$j = 7$

$$B_{87} = 2 \frac{Q}{V} F_1$$

$j = 8$

$$B_{88} = - 2 \frac{Q}{V} (F_2 + J_1) + B_{88}^{(f)}$$

$j = 9 \rightarrow 23$

$$B_{8j} = 0$$

$j = 24$

$$B_{8,24} = B_{8,24}^{(f)}$$
$$B_{8,25} = 0$$

Sloshing (i = 9 → 18):

$j = 1 \rightarrow 8$

$$B_{ij} = 0$$

$j = 9 \rightarrow 18$

$B_{ij} = 0$

$i \neq j$

$B_{ij} = 2\omega_{fi} \zeta_{fi}$

$i = j$

$j = 19 \rightarrow 25$

$B_{ij} = 0$

Rate Gyro (i = 19, 20):

$j = 1 \rightarrow 6$

$B_{ij} = Y'_{Rj} (X_{Ri})$

$j = 7$

$B_{i7} = 0$

$j = 8$

$B_{i8} = - 1$

$j = 9 \rightarrow 18$

$B_{ij} = 0$

$j = 19, 20$

$B_{ij} = 0$

$i \neq j$

$B_{ij} = 2\zeta_{Ri}/\omega_{Ri}$

$i = j$

$j = 21 \rightarrow 25$

$B_{ij} = 0$

Accelerometers (i = 21, 22):

$j = 1 \rightarrow 20$

$B_{ij} = 0$

$j = 21, 22$

$B_{ij} = 0$

$i \neq j$

$B_{ij} = 2\zeta_{Ai}/\omega_{Ai}$

$i = j$

$j = 23 \rightarrow 25$

$B_{ij} = 0$

Angle-of-Attack meter (i = 23):

$j = 1 \rightarrow 6$

$$B_{23j} = \frac{2\zeta_V}{\omega_V} \quad Y_j'(X_V) + \frac{1}{V} Y_j(X_V)$$

$j = 7$

$$B_{23,7} = \frac{1}{V}$$

$j = 8$

$$B_{23,8} = - \frac{2\zeta_V}{\omega_V} - \frac{\bar{X}_V}{V}$$

$j = 9 \rightarrow 22$

$$B_{23j} = 0$$

$j = 23$

$$B_{23,23} = 2/\omega_v \; (\zeta_v + \zeta_m)$$

$j = 24, 25$

$$B_{23j} = 0$$

Swivel Engine $(i = 24)$:

$j = 1 \rightarrow 23$

$$B_{24j} = 0$$

$j = 24$

$$B_{24,24} = - 2 \frac{\zeta_E}{\omega_E}$$

$j = 25$

$$B_{24,25} = 2 \; K_1 \frac{\zeta_E}{\omega_E}$$

The constant coefficients of d_{ij} are

Bending Modes $(i = 1 \rightarrow 6)$:

$j = 1 \rightarrow 6$

$$C_{ij} = Q \; C_{ij}^{(a)} - C_{ij}^{(f)} - S_E \; \bar{g} \; Y_i'(X_E) \; Y_j'(X_E)$$

$$- M_E \; \bar{g} \left[Y_j'(X_E) \; Y_i(X_E) + Y_i'(X_E) \; Y_j(X_E) \right]$$

$$- \omega_{Bi}^2 \; M_{Bi} \quad \text{when } i = j$$

$j = 7$

$$C_{i7} = 0$$

$j = 8$

$$C_{i8} = 2Q\ D_i + \bar{g}\left[S_E\ Y_i'(X_E) + M_E\ Y_i(X_E)\right]$$

$j = 9 \rightarrow 18$

$$C_{ij} = -\ Y_{fij}'\ (X_{fj})\ \bar{g}\ M_{fj}$$

$j = 19 \rightarrow 23$

$$C_{ij} = 0$$

$j = 24$

$$C_{i24} = F_s\ Y_i(X_E) + S_E\ \bar{g}\ Y_i'(X_E)$$

$j = 25$

$$C_{i25} = R\ Y_i(X_E)$$

<u>Translation (i = 7):</u>

$j = 1 \rightarrow 6$

$$C_{7j} = -\ Q\ E_j - (F - X)\ Y_j'(X_E) + C_{7j}^{(f)}$$

$j = 7$

$$C_{77} = 0$$

$j = 8$

$$C_{78} = 2Q\ F_o + (F - X) + C_{78}^{(f)}$$

$j = 9 \rightarrow 23$

$$C_{7j} = 0$$

$j = 24$

$$C_{7,24} = F_s$$

$j = 25$

$$C_{7,25} = R$$

Rotation (i = 8):

$j = 1 \rightarrow 6$

$$C_{8j} = - Q\bar{E}_j + \left[\bar{X}_E \, Y'_j(X_E) - Y_j(X_E) \right] (F - X) + C_{8j}^{(f)}$$
$$+ M_E \, \bar{g} \, Y(X_E) + S_E \, \bar{g} \, Y'_j(X_E)$$

$j = 7$

$$C_{87} = 0$$

$j = 8$

$$C_{88} = - 2Q \, F_1 - S_E \, \bar{g}$$

$j = 9 \rightarrow 18$

$$C_{8j} = M_{fj}$$

$j = 19 \rightarrow 23$

$$C_{8j} = 0$$

<u>Rate Gyro (i = 19, 20):</u>

$j = 1 \rightarrow 18$

$C_{ij} = 0$

$j = 19, 20$

$C_{ij} = 0$

$i \neq j$

$C_{ij} = 1$

$i = j$

$j = 21 \rightarrow 25$

$C_{ij} = 0$

<u>Accelerometers (i = 21, 22):</u>

$j = 1 \rightarrow 6$

$C_{ij} = - Y'_{Aj}(X_{Ai})\ \bar{g}$

$j = 7$

$C_{i7} = 0$

$j = 8$

$C_{i8} = \bar{g}$

$j = 9 \rightarrow 20$

$C_{ij} = 0$

$j = 24$

$$C_{8,24} = - \bar{X}_E F_S - S_E \bar{g}$$

$j = 25$

$$C_{8,25} = - \bar{X}_E R$$

<u>Sloshing ($i = 9 \rightarrow 18$):</u>

$j = 1 \rightarrow 6$

$$C_{ij} = \bar{g} Y'_{fj}(X_{fi})$$

$j = 7$

$$C_{i7} = 0$$

$j = 8$

$$C_{i8} = - \bar{g}$$

$j = 9 \rightarrow 18$

$$C_{ij} = 0$$

$$i \neq j$$

$$C_{ij} = \omega^2_{fi}$$

$$i = j$$

$j = 19 \rightarrow 25$

$$C_{ij} = 0$$

$j = 21, 22$

$C_{ij} = 0$

$i \neq j$

$C_{ij} = 1$

$i = j$

$j = 23 \rightarrow 25$

$C_{ij} = 0$

Angle-of-Attack meter (i = 23):

$j = 1 \rightarrow 6$

$C_{23j} = Y_j'(X_V)$

$j = 7$

$C_{23,7} = 0$

$j = 8$

$C_{23,8} = -1$

$j = 9 \rightarrow 22$

$C_{23j} = 0$

$j = 23$

$$C_{23,23} = \begin{cases} 1 \text{ if vane} \\ e^{i\lambda_V |s|} \text{ if local} \end{cases} \qquad |\,| = \text{absolute value}$$

$j = 24, 25$

$C_{23j} = 0$

Swivel Engine (i = 24):

$j = 1 \rightarrow 6$

$$C_{24j} = \frac{S_E\ \bar{g}}{\theta_E\ \omega_E^2} Y'_j(X_E)$$

$j = 7$

$$C_{24,7} = 0$$

$j = 8$

$$C_{24,8} = -\frac{S_E\ \bar{g}}{\theta_E\ \omega_E^2}$$

$j = 9 \rightarrow 23$

$$C_{24j} = 0$$

$j = 24$

$$C_{24,24} = -1 - \frac{S_E\ \bar{g}}{\theta_E\ \omega_E^2}$$

$j = 25$

$$C_{24,25} = K_1$$

Control Equation (i = 25):

$j = 1 \rightarrow 6$

$$d_{25j} = Y'_j(X_g)\ a_o T_g(s)\ A(s)\bar{K}_j e^{i\delta j}$$

$j = 7$

$$d_{25,7} = -\ A(s)\ \bar{K}_7 e^{i\delta_7}$$

$j = 8$

$$d_{25,8} = - T_g(s)\ a_o\ A(s)\ \bar{K}_8 e^{i\delta_8}$$

$j = 9 \rightarrow 18$

$$d_{25j} = 0$$

$j = 19 \rightarrow 20$

$$d_{25j} = -a_{1j}\ T_{Rj}(s)\ A(s)\ \bar{K}_j e^{i\delta_j}$$

$j = 21,\ 22$

$$d_{25j} = - g_{2j}\ T_{Aj}(s)\ A(s)\ \bar{K}_j e^{i\delta_j}$$

$j = 23$

$$d_{25,\ 23} = -b_o\ \bar{T}_{23}(s)\ A(s)\ \bar{K}_j\ e^{i\delta_j}$$

$j = 24$

$$d_{25,24} = 0$$

$j = 25$

$$d_{25,25} = \bar{K}_A e^{i\delta_A}$$

$\bar{K}_j$ will always be equal to 1 unless otherwise specified; δ_A and δ_j will always be equal to zero unless otherwise specified. These terms are included to facilitate the routine of phase and gain root locus studies.

Sloshing Parameters

$$\lambda_{nf} = \epsilon_n \frac{h_f}{a_f}$$

$$\omega^2_{nf} = \frac{\epsilon_n}{a_f} \bar{g} \tanh \lambda_{nf}$$

$$m_{sfn} = \frac{2 \tanh \lambda_{nf}}{\lambda_{nf} (\epsilon_n^2 - 1)} m_f$$

$$I_{corr} = \sum_{f=1}^{m} \frac{8m_f a_f^3}{h_f} \sum_{n=1}^{f} \frac{(\lambda_n - 2 \tanh \frac{\lambda_{nf}}{2})}{(\epsilon_n^2 - 1) \epsilon_n^3}$$

These equations for the sloshing parameters should be programmed so that the values can be computed in the program. Also provision should be made to load these values in.

The aerodynamic and fuel flow equations are given for reference but will not be programmed in the stability analysis program. Included are the print-out desired and a set of data sheets that are to be used for presenting data.

TABLE I. AERODYNAMIC COEFFICIENTS

Symbol	Slender Body Theory	Quasi-Steady Aerodynamics
$A_{ij}^{(a)}$	$\int_{-L/2}^{L/2} \lambda^2(X)Y_i(X)Y_j(X)$	
$B_{ij}^{(a)}$	$-2\int_{-L/2}^{L/2} Y_i(X)\left[\lambda\lambda'(X)Y_j(X)+\lambda^2(X)Y_j'(X)\right]dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_i(X)Y_j(X)dX + \frac{1}{4}C_{z\alpha f}Y_i(X_f)Y_j(X_f)$
$C_{ij}^{(a)}$	$-\int_{-L/2}^{L/2} Y_i(X)\left[2\lambda\lambda'(X)Y_j'(X)+\lambda^2(X)Y_j''(X)\right]dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_i(X)Y_j'(X)dX + \frac{1}{4}C_{z\alpha f}Y_i(X_f)Y_j'(X_f)$
$D_{i \text{ or } j}$	$\int_{-L/2}^{L/2} \lambda\lambda'(X)Y_{i \text{ or } j}(X)dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_j(X)dX + \frac{1}{4}C_{z\alpha f}Y_j(X_f)$
$\bar{D}_{i \text{ or } j}$	$\int_{-L/2}^{L/2} \lambda\lambda'(X)\bar{X}(X)Y_{i \text{ or } j}(X)dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_j(X)\bar{X}(X)dX + \frac{1}{4}C_{z\alpha f}Y_j'(X_f)$
E_i	$\int_{-L/2}^{L/2} \left[2\lambda\lambda'(X)Y_j'(X) + \lambda^2(X)Y_j''(X)\right]dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_j'(X)dX + \frac{1}{4}C_{z\alpha f}Y_j'(X_f)$
$\bar{E}_i$	$\int_{-L-2}^{L/2} \left[2\lambda\lambda'(X)Y_j(X) + \lambda^2(X)Y_j''(X)\right]\bar{X}dX$	$\frac{1}{4D_o}\int_{-L/2}^{L/2} C'_{z\alpha}(X)Y_j'(X)\bar{X}(X)dX + \frac{1}{4}C_{z\alpha f}Y_j'(X_f)\bar{X}(X)$

TABLE I (Cont'd)

Symbol	Slender Body Theory	Quasi-Steady Aerodynamics
F_k	$\int_{-L/2}^{L/2} \lambda\lambda'(X)\bar{X}^k \, dX \quad k = 0 \rightarrow 2$	$\frac{1}{4D_o} \int_{-L/2}^{L/2} C'_{z\alpha}(X) \, \bar{X}^k \, dX + \frac{1}{4} C_{z\alpha f} \bar{X}_f^k \quad k = 0, 1, 2$
$G_{i \text{ or } j}$	$\int_{-L/2}^{L/2} \lambda^2(X) \, Y_{i \text{ or } j}(X) \, dX$	
$\bar{G}_{i \text{ or } j}$	$\int_{-L/2}^{L/2} \lambda^2(X) \, \bar{X}(X) \, Y_{i \text{ or } j}(X) \, dX$	
H_j	$\int_{-L/2}^{L/2} \lambda^2(X) \, Y'_j(X) \, dX$	
$\bar{H}_j$	$\int_{-L/2}^{L/2} \lambda^2(X) \, \bar{X}(X) \, Y'_j(X) \, dX$	
J_k	$\int_{-L/2}^{L/2} \lambda^2(X) \, \bar{X}^k \, dX \quad k = 0 \rightarrow 2$	

TABLE II

FUEL FLOW

Symbol	(f denotes tank)
A_{jf} =	$\int_{X_1}^{X_2} \frac{Y''_j(X)\, dX}{A_f(X)}$
B_{jf} =	$\int_{X_1}^{X_2} \frac{\bar{X}(X)\; Y''_j(X)\, dX}{A_f(X)}$
C_{ijf} =	$\int_{X_1}^{X_2} \frac{Y_i(X)\; Y''_j(X)\, dX}{A_f}$
D_{if} =	$\int_{X_1}^{X_2} Y_i(X)\, dX$
E_{jf} =	$\int_{X_1}^{X_2} Y'_j(X)\, dX$
F_{ijf} =	$\int_{X_1}^{X_2} Y_i(X)\; Y'_j(X)$
G_{jf} =	$\int_{X_1}^{X_2} \bar{X}(X)\; Y'_j(X)\, dX$
H_f =	$\frac{1}{A_f(X_1)} - \frac{1}{A_f(X_2)}$
I_{jf} =	$\sum_{X_1}^{X_2} \frac{Y_j(X_n) + Y_j(X_{n+1})}{2} \left(\frac{1}{A_f(X_n)} - \frac{1}{A_s(X_{n+1})} \right)$

TABLE II (Cont'd)

Symbol		
J_{jf}	=	$\sum_{X_1}^{X_2} \frac{Y_j'(X_n) + Y_j'(X_{n+1})}{2} \quad \frac{1}{A_f(X_n)} \quad \frac{1}{A_f(X_{n+1})}$
K_{jk}	=	$\sum_{X_1}^{X_2} \frac{Y_j'(X_n)\ \bar{X}(X_n) + Y_j'(X_{n+1})\ \bar{X}(X_{n+1})}{2} \left(\frac{1}{A_f(X_n)} - \frac{1}{A_f(X_{n+1})}\right)$
$\bar{a}_f$	=	$\frac{\dot{m}_f^{\ 2}}{p_f}$
$\bar{b}$	=	$\lambda_E\ \dot{m}_E\ L_E$
$B_{ij}^{(f)}$	=	$\sum_{f=1}^{n} 2\dot{m}_f\ F_{ijf} + \bar{b}\left[2Y_j(X_E) + L_E\ Y_j'(X_E)\right] Y_i'(X_E)$
$B_{i8}^{(f)}$	=	$2\left[\sum_{f=1}^{n} \dot{m}_f\ D_{if} + \bar{b}\left\{Y_i(X_E) + \frac{L_E}{2}\ Y_j'(X_E)\right\}\right]$
$B_{i24}^{(f)}$	=	$2\bar{b}\left[Y_i(X_E) + \frac{L_E}{2}\ Y_i(X_E)\right]$
$B_{7j}^{(f)}$	=	$2\sum_{f=1}^{n} \dot{m}_f\ E_{jf} + 2\bar{b}\ Y_j'(X_E)$
$B_{78}^{(f)}$	=	$2\sum_{f=1}^{n} \dot{m}_f\ (X_{2f} - X_{1f}) + 2\bar{b}$

TABLE II (Cont'd)

Symbol		
$B_{8j}^{(f)}$	=	$2 \sum_{f=1}^{n} \dot{m}_f \; G_{jf} + \bar{b} \left[2\bar{X}_E + L_E \right] Y_j'(X_E)$
$B_{88}^{(f)}$	=	$2 \sum \dot{m}_f \; (X_{2f} \; X \; X_{1f}) \; \frac{(X_{2f} - X_{1f})}{2} + \bar{b} \; (\bar{X}_E + \frac{L_E}{2})$
$B_{8,24}^{(f)}$	=	$2\bar{b} \left[\bar{X}_E + \frac{L_E}{2} \right]$
$C_{ij}^{(f)}$	=	$\sum_{f=1}^{n} \bar{a}_f \; C_{ijf}$
$C_{7j}^{(f)}$	=	$\sum_{f=1}^{n} a_f \; (J_{jf} - A_{jf})$
$C_{78}^{(f)}$	=	$\sum_{f=1}^{n} \bar{a}_f \; H_f$
$C_{8j}^{(f)}$		$\sum_{f=1}^{n} \bar{a}_f \; (B_{jf} + I_{jf} - K_{jf})$

TABLE III

PRINT OUT DESIRED

	Real	Imaginary	Absolute	Phase
X_1				
X_2				
X_n				
$T_g(s)$				
$T_g(s)\ A(s)$				
T_{R19}				
$T_{R19}A(s)$				
T_{R20}				
$T_{R20}A(s)$				
T_{A21}				
$T_{A21}A(s)$				
T_{A22}				
$T_{A22}A(s)$				
$\bar{T}$				
$\bar{T}\ A(s)$				
$d_{n1}X_{j1}$				
$d_{n2}X_2$				
$d_{nn}X_n$				

TABLE III (Cont'd)

	Real	Imaginary	Absolute	Phase
$\varphi_i(X)$				
$A_i(X)$				

Root $s = \sigma + i\omega$

SLOSHING PARAMETERS AND BENDING DATA TIME____________

These data come on punched cards from another program or direct load-in.

i_f	$Y_1(x_f)$	$Y_2(x_f)$	$Y_3(x_f)$	$Y_4(x_f)$	$Y_5(x_f)$	$Y_6(x_f)$	$\bar{X}_f$	h_f	a_f
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
	$Y'(x_f)$	$Y'_1(x_f)$	$Y'_2(x_f)$	$Y'_3(x_f)$	$Y'_4(x_f)$	$Y'_5(x_f)$	m_f	ω_f	m_{sf}
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
	ζ_{sf}		ϵ_1		δ_{Bj}	ω_{Bi}	M_{Bi}		
1			ϵ_2						
2									
3									
4									
5									
6									
7									
8									
9									
10									

AERODYNAMIC AND CONTROL COEFFICIENTS

TIME__________

A_{ij}

$j \rightarrow$ / i	1	2	3	4	5	6	G_i	$\bar{G}_i$	D_i
1									
2									
3									
4									
5									
6									
			B_{ij}				$\bar{D}_i$	H_i	$\bar{H}_i$
1									
2									
3									
4									
5									
6									
			C_{ij}				E_i	$\bar{E}_i$	$Y'_j(X_v)$ $Y'_j(X_v)$
1									
2									
3									
4									
5									
6									
J_o		F_o			These data come on punched cards				
J_1		F_1			from another program or direct				
J_2		F_2			load-in.				
				Sensor Deflections					
	$Y(x_e)_j$	$Y'_j(x_e)$	$Y'_{rj}(19)$	$Y'_{rj}(20)$	$Y_{aj}(x_{21})$	$Y_{aj}(21)$	$Y_{aj}(22)$	$Y'_{aj}(22)$	$Y'_j(x\varphi)$
1									
2									
3									
4									
5									
6									

FUEL FLOW COEFFICIENTS

TIME________________

B_{ij}

$j \rightarrow$ / i	1	2	3	4	5	6	B_{i8}	B_{i24}	C_{7i}
1									
2									
3									
4									
5									
6									
			C_{ij}				B_{7j}	B_{8j}	C_{8j}
1									
2									
3									
4									
5									
6									
B_{78}									
B_{88}			These data come on punched cards						
B_{824}			from another program or direct						
C_{78}			load-in.						

CALCULATION OR DATA SHEET

$A(s)$	A_{ao}	A_{a1}	A_{a2}	A_{a3}	A_{a4}	A_{a5}	A_{a6}	A_{a7}	A_{a8}	A_{a9}	A_{a10}	Time	L_{r1}	L_{r2}	ω_{r1}	ω_{r2}	ω_v
	B_{ao}	B_{a1}	B_{a2}	B_{a3}	B_{a4}	B_{a5}	B_{a6}	B_{a7}	B_{a8}	B_{a9}	B_{a10}						
												Time	L_{A1}	L_{A2}	ω_{A1}	ω_{A2}	L_M
T_{R19}	a_{Ro}	a_{R1}	a_{R2}	a_{R3}	a_{R4}	a_{R5}	a_{R6}	a_{R7}	a_{R8}	a_{R9}	a_{R10}						
	b_{Ro}	b_{R1}	b_{R2}	b_{R3}	b_{R4}	b_{R5}	b_{R6}	b_{R7}	b_{R8}	b_{R9}	b_{R10}	Time	L_v	λ_v	L_E	ω_E	
T_{R20}	a_{Ro}	a_{R1}	a_{R2}	a_{R3}	a_{R4}	a_{R5}	a_{R6}	a_{R7}	a_{R8}	a_{R9}	a_{R10}						
												Time	λ_E	P_E	$\dot{M}_E$	$X_{1\dot{v}} \rightarrow$	X_{ff}
	b_{Ro}	b_{R1}	b_{R2}	b_{R3}	b_{R4}	b_{R5}	b_{R6}	b_{R7}	b_{R8}	b_{R9}	b_{R10}						
T_{A21}	a_{Ro}	a_{R1}	a_{R2}	a_{R3}	a_{R4}	a_{R5}	a_{R6}	a_{R7}	a_{R8}	a_{R9}	a_{R10}	Time	$X_{2i} \rightarrow$	X_{2f}			
	b_{Ro}	b_{R1}	b_{R2}	b_{R3}	b_{R4}	b_{R5}	b_{R6}	b_{R7}	b_{R8}	b_{R9}	b_{R10}						
T_{A22}	a_{Ro}	a_{R1}	a_{R2}	a_{R3}	a_{R4}	a_{R5}	a_{R6}	a_{R7}	a_{R8}	a_{R9}	a_{R10}						
	b_{Ro}	b_{R1}	b_{R2}	b_{R3}	b_{R4}	b_{R5}	b_{R6}	b_{R7}	b_{R8}	b_{R9}	b_{R10}						
$\bar{T}_{23}$	$\bar{a}_o$	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$\bar{a}_4$	$\bar{a}_5$	$\bar{a}_6$	$\bar{a}_7$	$\bar{a}_8$	$\bar{a}_9$	$\bar{a}_{10}$						
	$\bar{b}_o$	$\bar{b}_1$	$\bar{b}_2$	$\bar{b}_3$	$\bar{b}_4$	$\bar{b}_5$	$\bar{b}_6$	$\bar{b}_7$	$\bar{b}_8$	$\bar{b}_9$	$\bar{b}_{10}$						

MSFC - Form 799 (December 1960)

CALCULATION OR DATA SHEET

Time	a_0	a_{119}	a_{120}	Q	V	F	F_s	$\bar{g}$	g_{221}	g_{222}	I	m	m_e	L_e	θ_E	S_E	b_0
T_g (S) Time	a_{g0}	a_{g1}	a_{g2}	a_{g3}	a_{g4}	a_{g5}	a_{g6}	a_{g7}	a_{g8}	a_{g9}	a_{g10}	e_o	e_1	I_{corr}	$\bar{K}_1 \rightarrow$		$\bar{K}_j$
Time	b_{g0}	b_{g1}	b_{g2}	b_{g3}	b_{g4}	b_{g5}	b_{g6}	b_{g7}	b_{g8}	b_{g9}	b_{g10}	$\bar{K}_A$	R	$\bar{X}_R$	$\bar{X}_V$	$\bar{X}_{A21}$	$\bar{X}_{A22}$

MSFC - Form 799 (December 19[illegible])

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APPROVAL NASA TM X-53109

NUMERICAL PROCEDURES FOR STABILITY STUDIES

By Robert S. Ryan

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This document has also been reviewed and approved for technical accuracy.

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